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Second Term Examination

IIIrd Semester (B.Tech) November -2007

Paper code-ETMA-201

Subject-Applied Mathematics-III

TIME : 1 HOUR 30 MINUTES

MAX. MARKS : 30

NOTE :- Attempt Q.No. 1 and any two more questions. All questions carry equal marks.

1. (a) Form the partial differential equation by eliminating the arbitrary function from $f(xy + z^2, x + y + z) = 0$. 3
- (b) Prove that $\frac{d}{dx} [x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$ 3
- (c) Prove that $P_n(1) = 1$. 2
- (d) Prove that $\int_{-1}^1 P_n(x) dx = 0, n \neq 0$. 2
2. (a) Show that $\int_0^p x [ber^2(x) + bei^2(x)] dx = p [berp bei' p - beip ber' p]$ 4
- (b) If α and β are the roots of $J_n(x) = 0$, then prove that
- $$\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = \begin{cases} 0 & \text{if } \alpha \neq \beta \\ \frac{1}{2} J_{n+1}^2(\alpha) & \alpha = \beta \end{cases} \quad 6$$
3. (a) Prove that $\int_{-1}^1 x^2 P_{n+1}(x) P_{n-1}(x) dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$ 4
- (b) Prove that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} [(x^2 - 1)^n]$. 6
4. (a) Using the method of separation of variables, solve the partial differential equation $4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ given that $u(0, y) = 4e^{-y} - e^{-5y}$. 4
- (b) Solve the boundary value problem, $\frac{\partial^2 y}{\partial t^2} = 4 \frac{\partial^2 y}{\partial x^2}$, given that 6
- $$y(0, t) = 0, \quad y(5, t) = 0 \quad y(x, 0) = 0 \quad \text{and} \quad \left(\frac{\partial y}{\partial t} \right)_{t=0} = 5 \sin \pi x .$$